Please write clearly in block capitals.

Centre number | | | | | Candidate number | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
MATHEMATICS
Unit Further Pure 4

Wednesday 25 May 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The three points A, B and C have coordinates $(2,\ -1,\ 1)$, $(4,\ 3,\ -2)$ and $(3,\ 0,\ -3)$ respectively.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

[3 marks]

(b) Hence find the exact value of the area of triangle ABC.

[2 marks]

QUESTION PART REFERENCE

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2 The matrices $\mathbf{A}$ and $\mathbf{B}$ are such that

$$(\mathbf{AB})^{-1} = \begin{bmatrix} 10 & 3 \\ 2 & -1 \end{bmatrix} \quad \text{and} \quad \det(\mathbf{A}^{-1}) = \frac{1}{2}$$

Find the determinant of $\mathbf{B}$.

[3 marks]

QUESTION PART REFERENCE

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3 Let $\Delta = \begin{vmatrix} a & -12 & 1 \\ 4 & -3a & a-3 \\ -3 & 4a & a+4 \end{vmatrix}$.

(a) (i) Use a row operation to show that $a - 4$ is a factor of Δ.

[2 marks]

(ii) Factorise Δ as fully as possible.

[4 marks]

(b) The vectors $\begin{bmatrix} a \\ 4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -12 \\ -3a \\ 4a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ a-3 \\ a+4 \end{bmatrix}$ are linearly dependent.

Find the possible values of a.

[2 marks]

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4 The matrix $\mathbf{A} = \begin{bmatrix} 5 & 2 & 4 \\ 7 & 4 & 6 \\ 6 & k-2 & k \end{bmatrix}$ is non-singular.

(a) Show that $k \neq 5$.

[3 marks]

(b) Find $\mathbf{A}^{-1}$ in terms of k.

[5 marks]

(c) Use your result from part **(b)** to solve the equations

$$\begin{aligned} 5x + 2y + 4z &= a \\ 7x + 4y + 6z &= -a \\ 6x - 3y - z &= -5a \end{aligned}$$

[4 marks]

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5 The matrix $\mathbf{M} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 5 & -1 \\ 2 & p & 1 \end{bmatrix}$, where p is a constant, has three distinct eigenvalues, λ_1, λ_2 and λ_3.

(a) Given that $\lambda_1 = 2$, find the value of p.

[4 marks]

(b) Given that $\lambda_2 < 0$, find the values of λ_2 and λ_3.

[5 marks]

(c) The transformation T has matrix $\mathbf{M}$. Find a vector equation for any one of the invariant lines of T.

[2 marks]

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6 The transformation T maps (x, y) to (x', y') where

$$x' = -2x + 2y + 4$$
$$y' = 3x - y - 4$$

(a) Explain what is meant by a line of invariant points.

[1 mark]

(b) Find an equation of the line of invariant points of T.

[3 marks]

(c) Show that T has an infinite number of invariant lines.

[5 marks]

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7 The linear transformation T is represented by the matrix $\mathbf{M} = \begin{bmatrix} -k\sqrt{3} & 0 & -k \\ 0 & 2k & 0 \\ k & 0 & -k\sqrt{3} \end{bmatrix}$.

(a) Find the determinant of $\mathbf{M}$ in terms of k.

[1 mark]

(b) The transformation T is the composition of two transformations. The first transformation, T_1, is an enlargement centre O and scale factor $2k$ $(k > 0)$, and the second is a transformation T_2. When T is applied to a solid shape of volume $48\,\text{cm}^3$, the volume of the image is $0.75\,\text{cm}^3$.

(i) Find the value of k and hence state the scale factor of the enlargement.

[3 marks]

(ii) Give a full geometrical description of T_2.

[5 marks]

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8 (a) The plane Π_1 is perpendicular to the line $\left(\mathbf{r} - \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix}\right) \times \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

The point $(3,\ -1,\ 2)$ lies on Π_1. Find an equation for Π_1 in the form $\mathbf{r}.\mathbf{n} = c$.

[2 marks]

(b) The plane Π_2 contains the points $(2,\ 3,\ 2)$, $(4,\ 1,\ -1)$ and $(0,\ -1,\ 2)$.

Show that an equation for Π_2 is $2x - y + 2z = d$, where d is an integer to be found.

[5 marks]

(c) Show that the acute angle, θ, between the planes Π_1 and Π_2 is such that $\cos\theta = \frac{\sqrt{30}}{15}$.

[3 marks]

(d) The plane Π_3 has the equation $4(k-2)x + (k+1)y - 4k^2 z = 8$, where k is an integer. The three planes Π_1, Π_2 and Π_3 have no point in common.

(i) Show that one possible value of k is 1 and find the other possible value.

[4 marks]

(ii) State the geometrical configuration of the three planes for each of the two cases in part **(d)(i)**, giving reasons for your answers.

[4 marks]

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