A-LEVEL

Mathematics

Further Pure 4 – MFP4
Mark scheme

6360
June 2015

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\begin{vmatrix}1 & 3 & a\\ 2 & -4 & 7\\ 2 & 2 & -2\end{vmatrix} = 1\begin{vmatrix}-4 & 7\\ 2 & -2\end{vmatrix} - 2\begin{vmatrix}3 & a\\ 2 & -2\end{vmatrix} + 2\begin{vmatrix}3 & a\\ -4 & 7\end{vmatrix}$	M1		Correct expansion of triple scalar product
	or $(\mathbf{u}\times\mathbf{v}=)\begin{bmatrix}12\\ 4\\ -10\end{bmatrix}$			or correct vector product
	$=12a+48$	A1	2	**CAO**
(b)(i)	$12a+48=0$ $a=-4$	B1F	1	Sets their expression equal to 0 and solves the resulting linear equation correctly
(ii)	$\begin{bmatrix}1\\ 2\\ 2\end{bmatrix} = c\begin{bmatrix}3\\ -4\\ 2\end{bmatrix} + d\begin{bmatrix}-4\\ 7\\ -2\end{bmatrix}$ $c=3$	M1		Forming a system of equations and solving to correctly find either *c* or *d*
	$d=2$	A1		Both *c* and *d* correct
	$\mathbf{u}=3\mathbf{v}+2\mathbf{w}$	A1	3	**A1** Correct linear combination stated **NMS** $\mathbf{u}=3\mathbf{v}+2\mathbf{w}$ scores 3 marks
	Total		**6**	

Q2	Solution	Mark	Total	Comment
	(**a** + 2**b** – 6**c**) x (**a** – **b** + 3**c**)			
	= **a**x**a** – **a**x**b** +3**a**x**c** +2**b**x**a** – 2**b**x**b** +6**b**x**c** –6**c**x**a** +6**c**x**b** –18**c**x**c**	**M1**		Expansion of brackets – at least **<u>six</u>** terms correct with × or ∧
	= –**a**x**b**+3**a**x**c**+2**b**x**a**+6**b**x**c**-6**c**x**a**+6**c**x**b**	**A1**		Expansion fully correct unsimplified and use of **a**x**a** = **b**x**b** = **c**x**c** = **0** (seen or implied)
		m1		Use of **(a**x**b** = **–b**x**a** **or** **c**x**a** = **–a**x**c)** **and** **c**x**b** = **–b**x**c**
	= –9**c**x**a** + 3**b**x**a** or 3**j** + 3(–2**i**) + 2(3**j**) – 6(2**i**)			
	= –18**i** +9**j**	**A1,A1**	**5**	**A1** each term **Note** candidates who do not use vector product symbols eg $a^2 - ab + 3ac + ...$ or attempt to use components of vectors score **M0**
	Total		**5**	

Q3	Solution	Mark	Total	Comment
(a)	c_1 replaced by $c_1 + c_2$ gives $\begin{vmatrix} a+b-c & b-c & -bc \\ b+a-c & a-c & -ca \\ -c+a+b & a+b & ab \end{vmatrix}$	M1		Combining columns or rows sensibly, working towards first factor
	$(a+b-c)\begin{vmatrix} 1 & b-c & -bc \\ 1 & a-c & -ca \\ 1 & a+b & ab \end{vmatrix}$	A1		First factor correctly extracted
	r_2 replaced by $r_2 - r_1$ r_3 replaced by $r_3 - r_1$ $\begin{vmatrix} 1 & b-c & -bc \\ 0 & a-b & -ca+bc \\ 0 & a+c & ab+bc \end{vmatrix}$ $\begin{vmatrix} 1 & b-c & -bc \\ 0 & a-b & -c(a-b) \\ 0 & a+c & b(a+c) \end{vmatrix}$	m1		Combining rows or columns sensibly, working towards second factor
	$(a-b)(a+c)(a+b-c)\begin{vmatrix} 1 & b-c & -bc \\ 0 & 1 & -c \\ 0 & 1 & b \end{vmatrix}$	A1		**Three** factors correctly extracted and remaining determinant correct
	$\begin{vmatrix} 1 & b-c & -bc \\ 0 & 1 & -c \\ 0 & 1 & b \end{vmatrix} = b+c$	m1		Correct expansion to obtain final factor- dependent on previous **M1** and **m1**
	Hence full factorisation = $(a+b-c)(a-b)(a+c)(b+c)$	A1	6	Fully correct - **CSO**
(b)	Comparing gives $c=2$ and $b=3$	M1		Attempting to substitute $c=2$ and $b=3$ into their answer from part (a)
	Hence $5(a+1)(a-3)(a+2)\ (=0)$	A1F		Correct factors PI by correct values, provided FT is cubic equation in “a” with three linear factors
	$a=-2, -1, 3$	A1	3	**CSO** must have **6 marks** in part **(a)**
	Total		9	

Q3	Solution	Mark	Total	Comment
3	**ALTERNATIVE to (a)** r_2 replaced by $r_2 - r_1$ r_3 replaced by $r_3 - r_1$ $\begin{vmatrix} a & b-c & -bc \\ b-a & a-b & c(b-a) \\ -c-a & a+c & b(c+a) \end{vmatrix}$	(M1) (m1)		Combining columns or rows sensibly, working towards first factor Combining rows or columns sensibly, working towards second factor
	$(a-b)(a+c)\begin{vmatrix} a & b-c & -bc \\ -1 & 1 & -c \\ -1 & 1 & b \end{vmatrix}$	(A1)		First factor correctly extracted
	r_3 replaced by $r_3 - r_2$ $\begin{vmatrix} a & b-c & -bc \\ -1 & 1 & -c \\ 0 & 0 & b+c \end{vmatrix}$			
	$(b+c)\begin{vmatrix} a & b-c & -bc \\ -1 & 1 & -c \\ 0 & 0 & 1 \end{vmatrix}$	(A1)		Third factor correctly extracted
	$\begin{vmatrix} a & b-c & -bc \\ -1 & 1 & -c \\ 0 & 0 & 1 \end{vmatrix} = a+b-c$	(m1)		Correct expansion to obtain final factor- dependent on previous **M1** and **m1**
	Hence full factorisation = $(a+b-c)(a-b)(a+c)(b+c)$	(A1)	(6)	Fully correct – **CSO**
	ALTERNATIVE to (b) $\begin{vmatrix} a & 1 & -6 \\ 3 & a-2 & -2a \\ -2 & a+3 & 3a \end{vmatrix}$			
	$= a\begin{vmatrix} a-2 & -2a \\ a+3 & 3a \end{vmatrix} - 3\begin{vmatrix} 1 & -6 \\ a+3 & 3a \end{vmatrix} - 2\begin{vmatrix} 1 & -6 \\ a-2 & -2a \end{vmatrix}$	(M1)		Correctly expanding determinant
	$5(a+1)(a-3)(a+2) \quad (=0)$	(A1)		**CAO**
	$a = -2, -1, 3$	(A1)	(3)	**CSO**

Q4	Solution	Mark	Total	Comment
(a)	$\begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix} \quad (=0)$ $(1-\lambda)(4-\lambda)+2 \quad (=0)$	**M1**		
	$(\lambda-2)(\lambda-3) \quad (=0)$ $\lambda = 2, \quad 3$	**A1,A1**		**A1** each eigenvalue
	When $\lambda = 2$, $\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $x + y = 0$ OE	**M1**		Correct equation used to find eigenvector for either $\lambda = 2$ or $\lambda = 3$
	$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ or any multiple	**A1**		A correct eigenvalue found for $\lambda = 2$
	When $\lambda = 3$, $\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ or $2x + y = 0$ OE			
	$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ or any multiple	**A1**	**6**	A correct eigenvalue found for $\lambda = 3$
(b)	Using vectors above, required matrix columns must be multiples of $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ Comparing with $\begin{bmatrix} 4 & b \\ a & -2 \end{bmatrix}$	**M1**		Attempt to compare their eigenvectors with given matrix in correct order PI by correct value of a or b
	gives $a = -8$	**A1**		
	and $b = 2$	**A1**	**3**	**A1** each value
	Total		**9**	

Q5	Solution	Mark	Total	Comment
(a)	$\begin{bmatrix} 2 & -11 & -3 & 1 \\ 0 & 35 & 7 & 7 \\ 0 & 65 & 13 & 13 \end{bmatrix}$			from original $\begin{bmatrix} 2 & -11 & -3 & 1 \\ 5 & -10 & -4 & 6 \\ 9 & -17 & -7 & 11 \end{bmatrix}$
	or $\begin{bmatrix} 2 & -11 & -3 & 1 \\ 7 & 14 & 0 & 14 \\ 13 & 26 & 0 & 26 \end{bmatrix}$			
	or $\begin{bmatrix} 2 & -11 & -3 & 1 \\ 35 & 0 & -14 & 56 \\ 65 & 0 & -26 & 104 \end{bmatrix}$			
	Method 1 – row reduction to stage as above	M1		
		A1		Having row of 0s or stating one row is multiple of another
	Method 2 – elimination of one variable to obtain $35y+7z=7$ etc see above	(M1)		
	Two equations that are multiples of each other	(A1)		stating or showing one row is multiple of another or reducing both to same equ'n
	Let $y=\lambda$ Then	M1		Setting one variable equal to a parameter and obtaining expressions for **both** other variables
	$z=1-5\lambda$	A1		
	$x=2-2\lambda$	A1	5	**A1** each variable Other possibilities, eg $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8/5 \\ 1/5 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$; $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + \beta \begin{bmatrix} -4 \\ 2 \\ -10 \end{bmatrix}$
(b)	The equations represent three planes which meet in a line/form a sheaf.	E1	1	Must earn at least **two** marks in part (a)
	Total		**6**	

Q6	Solution	Mark	Total	Comment
(a)	$\begin{bmatrix}4\\1\\-2\end{bmatrix}-\begin{bmatrix}2\\1\\0\end{bmatrix}=\begin{bmatrix}2\\0\\-2\end{bmatrix}$ does not match direction ratios of line			Substituting and comparing with direction vector of line
	or $\begin{bmatrix}2\\0\\-2\end{bmatrix}\times\begin{bmatrix}3\\-1\\2\end{bmatrix}=\begin{bmatrix}-2\\-10\\-2\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\end{bmatrix}$	**B1**	**1**	or showing vector product is not zero
(b)	Direction vectors for plane are $\begin{bmatrix}3\\-1\\2\end{bmatrix}$, $\begin{bmatrix}2\\0\\-2\end{bmatrix}$	**B1**		Correct identification of one direction vector for plane, any multiples of these
	$\begin{bmatrix}3\\-1\\2\end{bmatrix}\times\begin{bmatrix}2\\0\\-2\end{bmatrix}$	**M1**		Both vectors correct (may have multiples, in any order) attempted
	$\begin{bmatrix}2\\10\\2\end{bmatrix}$	**A1**		Correct for their vector product Watch signs if terms in vector product are in a different order
	$c=\begin{bmatrix}2\\1\\0\end{bmatrix}.\begin{bmatrix}2\\10\\2\end{bmatrix}=14$	**M1**		Use of their normal vector and correct point eg (4,1,-2) to find value for *c*
	Plane is $\mathbf{r}.\begin{bmatrix}1\\5\\1\end{bmatrix}=7$ OE	**A1**	**5**	**CSO**

Q6 contd	Solution	Mark	Total	Comment
(c)	Equation of line perpendicular to plane and containing *D* is			
	$\mathbf{r} = \begin{bmatrix} 8 \\ -2 \\ 6 \end{bmatrix} + t\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$	M1		Equation of line through *D* using their normal from (b)
	Meets plane when (8 + *t*) +5(-2 +5*t*) +(6 +*t*)= 7	m1		Correct use of their line and their plane to obtain linear equation in *t*
	$t = \frac{1}{9}$	A1		Correct *t* value obtained
	Hence for reflected point, $t = 2 \times \frac{1}{9}$	B1F		Doubling their *t* value
	$\begin{bmatrix} 8 \\ -2 \\ 6 \end{bmatrix} + \frac{2}{9}\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix} =$ $\frac{1}{9}\begin{bmatrix} 74 \\ -8 \\ 56 \end{bmatrix}$	A1	5	Reflected point coordinates correct
	Total		**11**	

Q7	Solution	Mark	Total	Comment
(a)	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ gives $a+b=5$ $c+d=-3$	B1		both equations correct
	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ gives $a-b=1$ $c-d=-1$	B1		both equations correct
	Hence $a=3, b=2, c=-2, d=-1$	M1		Solving to get at least two correct values
	Matrix is $\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}$	A1	4	
(b)(i)	$\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}$	M1		Multiplication of matrices in correct order to form matrix equation - accept $\mathbf{TS} = \mathbf{A}$
	Hence $\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}^{-1}$	m1		Rearranging - correct order on RHS accept $\mathbf{T} = \mathbf{AS}^{-1}$
	$\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix}$	B1F		Correct inverse of their matrix from (a) seen anywhere
	$\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix}$	A1	4	**CAO**
	ALTERNATIVE $\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}$	(M1)		Multiplication of matrices in correct order to form matrix equation
	$\begin{bmatrix} 3p-2q & 2p-q \\ 3r-2s & 2r-s \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}$	(A1)		LHS fully correct
	$3p-2q = 3.4$ and $2p - q = 2$ Gives $p = 0.6$ and $q = -0.8$			
	$3r - 2s = 1.2$ and $2r - s = 1$ Gives $r = 0.8$ and $s = 0.6$	(A1)		Solving to find all correct values
	$\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix}$	(A1)	(4)	**CAO**
(ii)	(Anticlockwise) rotation	M1		Matrix must be correct in part **(b)(i)**
	through 53.1^0 (about O)	A1	2	Correct angle
	Total		10	

Q8	Solution	Mark	Total	Comment
(a)(i)	$\begin{vmatrix} 1 & 2 & k \\ 0 & 3 & 4 \\ -1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & k \\ 3 & 4 \end{vmatrix}$	M1		Correct expansion of 3 by 3 determinant
	$= -3 - 4 - 8 + 3k$	A1		Correct unsimplified, brackets removed
	$(= -15 + 3k)$			
	$k \neq 5$	A1	3	Correct conclusion
(ii)	$\begin{bmatrix} -7 & -4 & 3 \\ k+2 & k-1 & -3 \\ 8-3k & -4 & 3 \end{bmatrix}$	M1		one row or column correct
		A2		**A1** at least six terms correct **A2** all correct
	$\begin{bmatrix} -7 & k+2 & 8-3k \\ -4 & k-1 & -4 \\ 3 & -3 & 3 \end{bmatrix}$	m1		Transpose of their matrix – dependent on previous **M1**
	$\mathbf{M}^{-1} = \dfrac{1}{3k-15}\begin{bmatrix} -7 & k+2 & 8-3k \\ -4 & k-1 & -4 \\ 3 & -3 & 3 \end{bmatrix}$	A1	5	Fully correct
(b)	When $k = 1$, determinant of **M** = –12	B1		or det $\mathbf{M}^{-1} = -1/12$
	Hence volume scale factor = $\dfrac{1}{12}$			
	Image volume = $\dfrac{1}{12} \times 6$	M1		Correct use of $\pm$"*their*" volume scale factor to find image volume
	= 0.5 (cm^3)	A1	3	**CAO** – must be positive
(c)	$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & 4 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+2y+5z \\ 3y+4z \\ -x+y-z \end{bmatrix}$	M1		**M1** - Substituting $k = 5$ and multiplying - at least two components correct
		A1		**A1** all correct
	$x' - y' + z'$ $= (x+2y+5z) - (3y+4z) + (-x+y-z)$ $= 0$ Therefore each point lies in the plane $x - y + z = 0$	A1	3	**AG** be convinced Must see either first three lines or concluding statement when top line is missing

Q8cont'd	Solution	Mark	Total	Comment
(d)	$\begin{bmatrix} 1 & 2 & k \\ 0 & 3 & 4 \\ -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$			
	$\begin{bmatrix} x+2y+kz \\ 3y+4z \\ -x+y-z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	**M1**		Use of **Mv** = **v** with at least two "equations" correct
	$2y+kz=0$ $2y+4z=0$ $-x+y-2z=0$	**A1**		Fully correct with terms combined
	Hence $k=4$	**A1**		
	Equation of line is	**m1**		Using their equations to obtain Cartesian equations of line
	$\dfrac{x}{-4} = \dfrac{y}{-2} = z$ OE	**A1**	**5**	**CSO**
	Total		**19**	
	TOTAL		**75**	