Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
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6	
7	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MFP3

Unit Further Pure 3

Wednesday 13 May 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 It is given that $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = \frac{x + y^2}{x}$$

and $$y(2) = 5$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

with $h = 0.05$, to obtain an approximation to $y(2.05)$.

[2 marks]

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r)$$

with your answer to part **(a)**, to obtain an approximation to $y(2.1)$, giving your answer to three significant figures.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 1

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QUESTION PART REFERENCE	Answer space for question 1

2 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + (\tan x)y = \tan^3 x \sec x$$

given that $y = 2$ when $x = \frac{\pi}{3}$.

[9 marks]

QUESTION PART REFERENCE

Answer space for question 2

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QUESTION PART REFERENCE	**Answer space for question 2**

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3 (a) (i) Write down the expansion of $\ln(1+2x)$ in ascending powers of x up to and including the term in x^4.

[1 mark]

(ii) Hence, or otherwise, find the first two non-zero terms in the expansion of

$$\ln[(1+2x)(1-2x)]$$

in ascending powers of x and state the range of values of x for which the expansion is valid.

[3 marks]

(b) Find $\lim_{x \to 0} \left[\dfrac{3x - x\sqrt{9+x}}{\ln[(1+2x)(1-2x)]} \right]$.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

4 (a) Explain why $\int_2^{\infty} (x-2)\mathrm{e}^{-2x}\,\mathrm{d}x$ is an improper integral.

[1 mark]

(b) Evaluate $\int_2^{\infty} (x-2)\mathrm{e}^{-2x}\,\mathrm{d}x$, showing the limiting process used.

[6 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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5 (a) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 6\frac{\mathrm{d}y}{\mathrm{d}x} + 9y = 36\sin 3x$$

[7 marks]

(b) It is given that $y = \mathrm{f}(x)$ is the solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 6\frac{\mathrm{d}y}{\mathrm{d}x} + 9y = 36\sin 3x$$

such that $\mathrm{f}(0) = 0$ and $\mathrm{f}'(0) = 0$.

(i) Show that $\mathrm{f}''(0) = 0$.

[1 mark]

(ii) Find the first two non-zero terms in the expansion, in ascending powers of x, of $\mathrm{f}(x)$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE	Answer space for question 5

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6 A differential equation is given by

$$4\sqrt{x^5}\,\frac{d^2y}{dx^2} + (2\sqrt{x})y = \sqrt{x}(\ln x)^2 + 5, \quad x > 0$$

(a) Show that the substitution $x = e^{2t}$ transforms this differential equation into

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 2y = 4t^2 + 5e^{-t}$$

[7 marks]

(b) Hence find the general solution of the differential equation

$$4\sqrt{x^5}\,\frac{d^2y}{dx^2} + (2\sqrt{x})y = \sqrt{x}(\ln x)^2 + 5, \quad x > 0$$

[10 marks]

QUESTION PART REFERENCE

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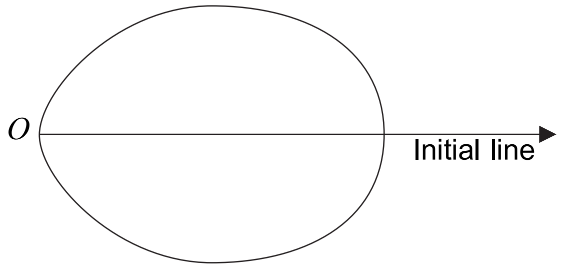
7 The diagram shows the sketch of a curve C_1.



The polar equation of the curve C_1 is

$$r = 1 + \cos 2\theta, \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

(a) Find the area of the region bounded by the curve C_1.

[5 marks]

(b) The curve C_2 whose polar equation is

$$r = 1 + \sin \theta, \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

intersects the curve C_1 at the pole O and at the point A. The straight line drawn through A parallel to the initial line intersects C_1 again at the point B.

(i) Find the polar coordinates of A.

[4 marks]

(ii) Show that the length of OB is $\frac{1}{4}\left(\sqrt{13} + 1\right)$.

[6 marks]

(iii) Find the length of AB, giving your answer to three significant figures.

[3 marks]

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END OF QUESTIONS

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